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RAILWAY RIGHT-OF-WAY SURVEYING

Albert I. Frye

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RAILWAY RIGHT-OF-WAY SURVEYING

BY

ALBERT I. FRYE, S.B., M. AM. SOC. C. E.



NEW YORK
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PREFACE.



THIS little book outlines a modern system of right-of-way surveying, leveling, and mapping. Though the system as a whole applies particularly to railroads, the "adjustment diagram" feature will be found useful in the special adjustment of streets, canals, highways, and old boundary lines. It is hoped the suggestions here given will save time in the organization and prosecution of the work and above all will lead to accurate and permanent results.

The method of survey herein described must not be confused with the common one of using the outside head of rail as a base line for locating the various structural features. Vast sums of money have been spent for such surveys, leaving no permanent field reference to lines run and recording sets of maps without any measurements whatever. Many of them have been ordered under the directing influence of rushing some kind of survey through as quickly as possible for purposes of reconstruction, hence no criticism is here offered.

But there is no excuse whatever for property surveys not referable to fixed base lines, whether they result in the "scale maps" above referred to or in the so-called "very complete (?) maps," showing instrument lines run, angles and measurements, "direct from the field book." The former class of maps is of little practical value, while the latter contains a mass of data not directly available.

The simplest system, and that giving the most accurate, available, and permanent records, is one that reduces the important field measurements and office data to rectangular coordinates referred to an established center line. This is the system here presented.

Without some definite plan of rearranging and tabulating data as obtained, much time will be lost and the progress of the work greatly

delayed. For this purpose the Journal and Ledger system herein described is invaluable and should be started as soon as possible.

It goes without saying that in this as in all work where great accuracy and permanent records are required, progress is slow. But the results as they appear will be lasting.

THE AUTHOR.

NEW YORK, September, 1904.

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INTRODUCTION.



WHEN a railroad line is located the real-estate agent is furnished with maps more or less complete but deemed sufficient for his immediate needs in purchasing right of way. Many if not all of those old maps have been lost.

Track is laid, narrow, broad, or standard gage, or changed from one to the other by changing one rail without shifting track to center. Perhaps the original single track was laid on the old center line, or perhaps on one side with a view to double-tracking later. The original line may have been run by "the needle" in a serpentine manner with spasmodic attempts later on to straighten it in places, leaving the right-of-way lines anything but certain, or the line changed altogether without any record of same.

The original stationing, in any case, becomes problematical, the original points having long since disappeared, and, further to complicate matters, the old chain was too long or too short by several inches.

Right-of-way fences have been built, at times, inconsistent with the true center line, producing jogs and bends here and there in the side lines intended to be continuous and regular.

The Real-estate Department wrestles with these problems, falls back upon the Law Department to interpret the various inconsistencies, and the latter finally concludes that too much time is being wasted for want of sufficient accurate data. It is decided that the cheapest and best way is to have a survey made, comprehensive in character, in order to straighten out the numerous land complications.

This is a wise decision. The general manager is informed that in a short time the survey will be found to produce economic results; that is, any cost of such work is economy over existing methods.

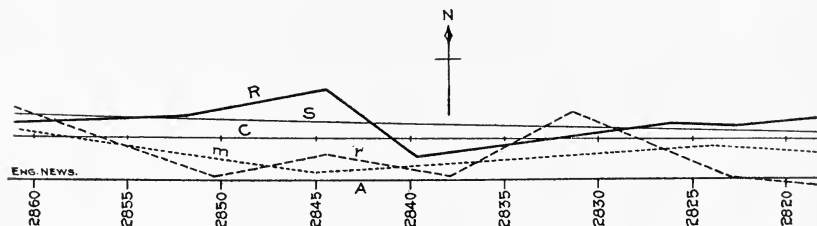
The following article appeared in *Engineering News* of April 14, 1904, and is inserted here as introductory to the general discussion:

PRACTICE IN RE-ESTABLISHING AND MONUMENTING A RAILWAY CENTER LINE.

By Albert I. Frye, M.Am.Soc.C.E.

The value of re-establishing and monumenting railroad center lines is recognized by the legal, real-estate, and engineering departments of Eastern railroads as an economic necessity. It crystallizes and harmonizes the actions of local surveyors in their property surveys adjacent thereto, and is a fitting preliminary to side-monumenting, adjustment of track, and various operations incident to new work.

The writer does not wish to enter into a general dissertation upon this subject, but merely to bring out one salient feature which has proved convenient and effective in adjusting the center lines of tangents.



ADJUSTMENT DIAGRAM.

An "instrument line" is run, say, 7 feet north of and parallel with an "assumed center line" of right of way, and all monumental landmarks, such as old monuments, fences, track centers, centers of structures, buildings, *etc.*, which may be *reasonably supposed* to have been placed at *certain* distances from the old center line, are located. The *ranges* and *stations* (rectangular coordinates) of these points are calculated with reference to the "assumed center line."¹

On a longitudinal scale of, say, 800 feet to an inch the "assumed center line" is laid off, the lateral scale being 1 foot to 1 inch. The various points which have been located and calculated are then platted from the "assumed center line," the lateral distance being the difference between the range as calculated and its *supposed* original distance from the old center line. For instance, if the range of an old fence were found to be 49.75 feet north of assumed center line, whereas it was supposed to have been placed 50 feet distant, then the fence at that point would tend to establish the center line 0.25 foot south of the "assumed" and the point would be platted "range 0.25 south."

It is essential to classify the various landmarks used and further to distinguish those found on the north side of right of way from those on the south, *connecting* each class on either side of right of way by its characteristic (colored) line.

¹ See page 14.

The following notation has been used with success:

A. (black line)	=	Assumed center line.	
R. (full brown line)	=		Railroad fences on north side of right of way.
r. (dotted brown line)	=	Center line as would be es- tablished by	Railroad fences on south side of right of way.
P. (full yellow line)	=		Private fences on north side of right of way.
p. (dotted yellow line)	=		Private fences on south side of right of way.
M. (full blue line)	=		Monuments on north side of right of way.
m. (dotted blue line)	=		Monuments on south side of right of way.
S. (full green line)	=		Existing structures.
T. (full pencil line)	=		Existing tracks.
C. (full red line)	=	Adjusted center line.	

The accompanying simple sketch will serve to illustrate.

The adjusted center line is best perfected (after all the others are platted) by the use of a fine thread, and in a manner similar to establishing grades on profiles. The lateral scale adopted—1 inch=1 foot—insures great accuracy. It must be remembered that all the lines shown are actual possible center lines crossing and recrossing each other in a maze of network and are in no sense offset lines, hence the final adjustment is a direct one.

The object in distinguishing the north monumental points from those on the opposite side of right of way is to avoid encroachment.

As frequently happens with long "tangents" run in the early days, the re-adjustment necessitates a slight "bend" the position of which should be fixed at some "round" station by a stone monument. The semi-graphical method above described readily points to the position of such a bend, and leaves a permanent record of existing facts and conclusions drawn. The center line being thus fixed is permanently monumented on the ground.

RAILWAY RIGHT-OF-WAY SURVEYING.

COLLECTING THE DATA.

BEFORE starting in the field it will be well to become familiar in a general way with the nature and amount of work to be accomplished, as well as the data at hand and necessary to be obtained in order to accomplish it. These data should be classified and indexed systematically, care being taken to record the date of each document or item, as much of its value may depend upon the year or even the month of its origin.

A careful, systematic search should be made in the several offices of the company for note-books, plans, letters, papers, or documents of any kind which supposedly bear upon the problem of fixing the center line or adjusting the various land lines. Sometimes a paper or letter relating almost wholly to construction matters will aid materially in clearing up the adjustment of a tangent.

City and county records must be examined for deeds and maps. If a former county has been divided, this fact must be noted in the searching. Copies of all city and tax maps should be had.

Much valuable information will be gathered along the line from surveyors and others incidentally as the work proceeds, and a diary or note-book should be kept for this purpose.

Prior to indexing, a *tentative* alinement sheet should be made up from the filed location of the line or from any authoritative data, numbering the tangents consecutively from the beginning of the line or station zero of the new survey. Of course this will be superseded by the regular "Alinement Sheet (A)" of the ledger, described later.

TENTATIVE ALINEMENT SHEET.

The tentative alinement sheet will be compiled from existing records and will represent, merely, the *old* and *new* theoretical alinements, preliminary in character, to serve as an aid to indexing and for both field and office reference. Generally, the new alinement should follow the center line as it will be monumented later, while the old will usually be the line as filed. Even if the two be not identical, the former should, preferably, be the center of the present 100 feet right of way. There may be exceptions to this rule, however, depending somewhat upon the position of the track.

The sheet should contain information as per following headings of *columns*:

New alinement.	{	1. Tangent No.
		2. Nearest depot.
		3. Approximate location.
		4. New stationing.
		5. Point, P.C. or P.T.
		6. Length of curve in feet.
		7. Degree of curve.
		8. Tangent distance (semi-tangent).
		9. Intersection angle (I).
		10. Length of tangent in feet.
		11. Length of tangent in miles and tenths.
		12. True course of tangent.
		13. Monument to be set.
		14. Distance to old center line N. or S.
Old alinement.	{	15. Old stationing of new monument points.
		16. Old stationing of old P.C. or P.T.
		17. New stationing of old P.C. or P.T.
		18. Length of old curve in feet.
		19. Old intersection angle (I).
		20. Tangent distance (semi-tangent).
		21. Length of old tangent in feet.
		22. Reference to notes, maps, etc.
		23. Remarks.

Note.—The *line* items will be for new P.C.'s, P.T.'s, and monuments to be set on the new center line. Monuments should be set at "round stations" from 1000 to 2000 feet apart on tangents and from 500 to 800 feet apart on curves, also at beginning and end of curves or, if impracticable, near same for reference. For Column No. 13 the kind of monument will be noted as "Stone" for stone monument, etc. The equation between the old and new stationing at these points may be obtained from 4 and 15.

INDEXING THE DATA.

FIELD BOOKS, especially old ones, and particularly those of a semi-private character if any exist, should be carefully examined for information regarding the center line, any monuments which may have been set or located, land surveys, etc.

They should be indexed in detail, by pages, on a general index sheet, referring to the numbered tangents as per the tentative alignment sheet, and in such a manner that they may readily be selected for field or office work.

The following system has been found well adapted for such reference:

INDEX TO NOTE-BOOKS.

Tang. No.	Description.	Date.	Book.	Pg.	Special Use.		Remarks.
					Field.	Office.	
6	Correct notes of Slater property.	4-24-71	25K	37	Center line	Maps	Valuable
4	Monument 350 ft. E. of Ave. G.	Aug., 1877	32M	8			
47	Land lines, B & S. crossing.	3-8-92	41P	82		Maps	

Similarly, the old maps should be indexed under a new system of numbers, but also retaining the old in the index book. The number will comprise the number of the tangent to which the map belongs by location, with a distinguishing letter suffixed.

The plans are classed according to tangent numbers both as regards indexing and filing. The numbers are suspended from the plans on tags, thus allowing the direct selection of plans pertaining to any tangent without being obliged, necessarily, to resort to the index.

The following simple system will illustrate:

INDEX TO MAPS.

Tang. No.	Description.	New No.	Scale.	Date.	Plan— Kind.	Old No.	Remarks.
4	Old line of R.R. at D.	4n	100	5-8-68	Trac.	C 16	
19 {	Canal crossing. . . .						
	Burgess property at	19b	80	4-3-85	{ Blue-	{ E 6	{ Valuable
	Clifton.				print		for field

OLD PAPERS.

Papers which have been in the archives of the company for years undisturbed and of ancient date, also loose notes of positions of old monuments and corners, or any information relating to land lines or lines in general which may appear to have value, may be placed in envelopes and numbered by tangents as above.

Sometimes a long tangent may require several envelopes and it may be more convenient to classify or even index the papers either as regards position on the tangent or according to the class of data which the paper contains.



SEARCHING DEEDS AND OTHER RECORDS.

ONE of the most important operations is that of searching. Abstracts of all deeds should be made in duplicate, one for the field and one for the drafting-room. These sheets may be of fairly large size, say 6"×8", typewritten, and with holes at the top for filing.

First come the headings, next the description, and lastly the special clauses, thus:

SAMPLE ABSTRACT SHEET.

Tangent No.	City or town	County
Date.	Map No.	Record Book, pg.
Kind of Deed.	Office Rec. Bk., pg.	Office No. of Deed
Grantor.	Grantee	
Description.		
Special clauses.		

FIELD MAPS.

In connection with the field copies of abstracts there should be accompanying maps, on tracing-cloth, of each tangent with included portions of adjacent curves. These maps may be drawn to a small scale for convenience, say 200 feet to an inch, showing streets, company properties with the map number of each lot as per abstract, all vital measurements, such as width of right of way, change in center line, positions of any monuments referred to in deeds or other records, and in fact any "map information" which may be useful in the field.

Where reference is made to monuments they should be hunted up in the field in advance of the regular field work.

STANDARD TAPE.

A 100-foot steel tape graduated to hundredths of a foot, properly certified to as correct as a certain temperature, say 60° , and at a certain tension, say 10 pounds when entirely supported, should be kept in the safe absolutely free from field work and used as a test tape only.

It is far preferable to a standard base with fixed ends in that no temperature correction is necessary, and it only remains to give both tapes, the standard and the one being tested, the same pull in order to eliminate tension as well as temperature correction.

Of course this does not do away with the use of the spring-balance or thermometer in the field where great accuracy is required, as, for instance, measuring long base lines, making city surveys or surveys for legal purposes.

It is well to keep in mind that a change in temperature of about 15° , or tension of 6 to 10 pounds (depending upon the area of cross-section of tape), will produce a change in length of $\frac{1}{100}$ of a foot in a 100-foot steel tape; hence for long measurements a considerable error might accumulate if left uncorrected.

FIELD INSTRUMENTS.

ALL field instruments should be first class.

The steel tapes should be graduated throughout to hundredths. The 100-foot tapes should be detachable from the box and be provided with handles for both ends. As they will be used considerably in measuring long base lines, the zero mark should come at the end of the steel ribbon itself and not at the end of an uncertain brass ring. The 50-foot tapes, for minor measurements, may have the brass ring if preferred.

The transits should be heavy and be provided with powerful telescopes if long tangents are to be run. A 12-ounce plumb-bob is not too heavy.

Flag-poles of the ordinary kind, both wood and iron, are generally convenient, but for running long tangents flag-boards, say 6 inches wide and provided with iron shoes properly centered, are necessary, or even at times a board 10 or 12 inches wide is called for. The latter need not be shod, however. The boards should be checkered red and white alternately, with well-defined vertical center lines, the horizontal lines dividing the colors being spaced about the width of the board apart. Such boards can be plainly sighted on clear days "with the sun" about $1\frac{1}{2}$ to $2\frac{1}{2}$ miles.

Standard survey-tacks with a depression in the middle are far preferable to the ordinary and should be used for instrument or flag-pole points.

Yellow crayon is the best for marking.

FIELD WORK.

THE PRACTICAL CENTER LINE.

IN re-establishing a railway center line it is essential to preserve in the main the line recognized as existing on the ground, evidenced by monuments, fences, buildings, bridges, tracks, stakes, etc. As in every problem there is a theoretical and a practical side, so in this the filed location is the theoretical, while the existing landmarks may be said to determine the practical location. The practical location governs.

Generally it will be necessary only to establish the center lines of two adjacent tangents, connecting them with the theoretical curve of filed location. If this latter, however, does not reasonably fit the practical curve on the ground as evidenced by controlling landmarks, then it may be necessary to alter the radius or degree of the curve accordingly, especially if it is a very long one. Inaccuracy in former field work, long preserved and uncorrected, as when right-of-way fences are being built, would allow the practical to take precedence.

Of course any radical departure from curve of filed location should be avoided if possible, and this may best be effected in some cases by a slight alteration of one or both of the adjacent tangents.

FIELD ADJUSTMENT OF CENTER LINE.

In many cases the center line can be established directly in the field, and this should be done where practicable, as the instrument line will then be a parallel offset to the true center line, thus simplifying or avoiding many office calculations.

A short tangent with few controlling landmarks, but definite and consistent in themselves, offers a favorable illustration for field adjustment. Points on the true center line or on an offset line are set off from the landmarks by knowing their assumed positions, and an instrument line run directly in the field along "these points," correcting any inconsistencies and giving due weight to the most important.

If this cannot be done, then a center line may be assumed as nearly as possible and an instrument line run parallel with it, the various

landmarks located and the line adjusted in the office as explained in the Introduction.

FIELD WORK FOR OFFICE ADJUSTMENT.

The instrument line may be run in the middle of one of the outside main tracks for safety and so that the various points on the line may be established by "tacks in ties" instead of by driving stakes. By experience it has been found that track ties, when well ballasted, will hold their position fairly well and much better than ordinary stakes, especially during periods of frost, with the added advantage of rapid work and a level support for measuring along the instrument or base line.

Of course track foremen must be notified not to tamp, shift, or in any manner disturb the track or ties during the operation.

Running the Base Line.—If the tangent is not too long for a distinct sight from one end to the other, the line may be run by "following up" with the transit, with the sun on the foresight, fixing well-centered points from 1000 to 2000 feet apart. Preferably these points should be opposite or near points to be monumented on the center line, the monuments for which have *previously* been distributed on the ground.

For long tangents it is well to begin in the middle, establishing two points as far apart as practicable, and work in either direction toward the ends by reversing the instrument. The method of reversing may be "four sights to right," then turn half-round and "four sights to left," preserving the order of reversing, infallibly.

The engineer in charge may be with the front flag and be provided with a field-glass to catch signals from the transit. If the instrument be out of adjustment, the true point will be midway between the average of each set of points provided they be given equal weight.

A signal for repetition of right or left sights may be called for by the front flagman by dropping his handkerchief intermittently to the right or left side the required number of times. This motioning will not be confused with the "general wave signal" for calling up.

An extra man for transmitting signals or messages is sometimes desirable, and the back flagman may also be provided with a field-glass or engineer's level to watch for signals.

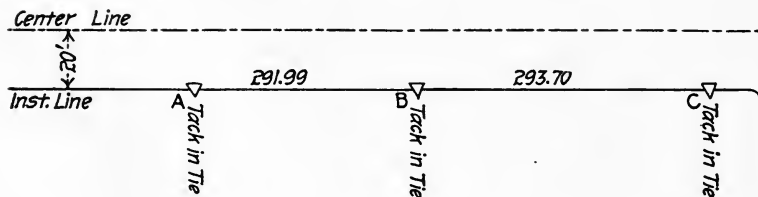
Referencing Instrument Points.—Points on the instrument line near center-line stations to be monumented should be referenced very substantially, soon after the line is run and prior to the general field work, as they will be used later in setting the monuments. The reference should be by at least two lateral measurements to preserve the

line, which is most important, and by two or more diagonal measurements. Good sound ties should be selected for these reference points, well marked with paint or crayon, and the matter explained to the section foreman.

Measuring the Base Line.—Intermediate points will be set on the base line a little less than 300 feet apart, so three measurements of the 100-foot tape will stretch between. They should be marked by standard tacks accurately centered for line, to be used later as transit points for lateral field work. These, together with those referenced near monument points, should be lettered consecutively, *A, B, . . . Y, Z; AA, AB, . . . AY, AZ; BA, BB, . . .* etc., throughout the tangent, on the ties, and on near-by telegraph poles, buildings, and fences along the right of way. The same lettering is of course recorded in the field book. The tacks in ties should also be plainly marked with a heavy surrounding triangle of yellow crayon.

The method of measuring the base line and keeping the notes must be systematic and very exact. A very simple system is to measure from one "300-foot" point to the next and then back, keeping the measurements in two columns, taking the average of the results, making the proper temperature correction, and putting the final corrected result on the line itself, *immediately*, thus:

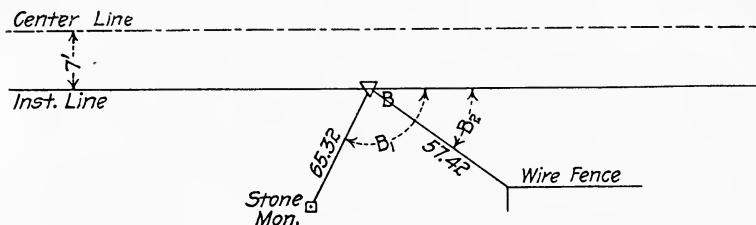
<i>A to B.</i>		<i>B to C.</i>	
97.795	97.796	99.832	99.836
99.632	99.631	99.524	99.525
94.538	94.536	94.316	94.313
<u>291.965</u>	<u>291.963</u>	<u>293.672</u>	<u>293.674</u>
291.964		293.673	
Temp. corr. .024		Temp. corr. .025	
<u>291.988</u>		<u>293.698</u>	



Lateral Field Work.—From these points *A, B, C*, etc., on the base line, the side monumental features are located. As with the base line itself, all measurements should be taken and recorded in the same manner and the corrected measurements set down immediately on

the lines measured. Angles should be at least doubled, as a check, no matter how close the object.

A good system is to set down the angles to the object *above the measurements* to same, and the corrected measurements on the sketch itself. The angles to objects may be designated by the letter name of the instrument point on the base line, with the suffixes "sub 1," "sub 2," etc., where more than one angle is turned from the same point, thus:



B_1		B_2	
112-32-30		52-16-00	
225-05-00		104-32-30	
Call 112-32-30		Call 52-16-15	
65.327	65.329	57.426	57.423
65.328		57.424	
Temp. corr. .005	Temp. corr. .	.004	
65.323		57.420	

Note.—The length of tape should be recorded on each page, as, for instance, "Tape correct at 63° and 10 lbs." ¹ As tapes are often exchanged, this becomes very necessary. Also the field temperature of the ground must be recorded for each page, and often for each line measured, as the case may demand.

Base-line Angles.—Angles in the instrument or base lines, as chords connecting tangents, should be measured by ten continuous readings, and these must be examined, by subtraction, for "slip" before dividing the final reading by its number.

Checking the Field Notes.—The field-book notes should be checked as soon as possible, using a check-mark with the initial of the checker therein; thus, \checkmark^H means, checked by "H," or Harris.

The check should be after every item and not simply once at the bottom of the page.

¹ Tension.

BOOKKEEPING OF THE SURVEY.

IN a survey of this character it would be the height of folly to plat the field notes directly on the maps "just as they are taken in the field." The author has seen such work and heard arguments put forth in its favor, but has always discouraged it.

The field notes are merely a means to an end and not the end itself. The field book corresponds to the day book of a business firm and should have an accompanying journal and a ledger. Imagine an accountant, at the end of the year, exhibiting his day book to the manager as a clear and concise statement of the firm's business! Yet this is a fair illustration of the above.

In addition to the journal and ledger, a monument book should be kept, as will be explained later.

THE JOURNAL.

A very convenient system of office record is to have the line, as it is developed by the survey, platted in a large cross-section book, or journal, the pages large enough to embrace (on a reduced scale) the outline of the right-of-way map.

For instance, suppose we wish to adopt a 50-foot scale for the maps themselves, making them, say, 28"×42" inside of border line and leaving a $\frac{1}{2}$ -inch margin all round. This will economically use 58" Paragon mounted paper, which is really 59" wide.

Description.—A handy journal for such size maps, and one which has been found satisfactory, is as follows: Pages 10.7"×16.4", with cross-section lines spaced 5.8 to the inch. Using a platting scale of one space to 25 feet, four spaces per 100 feet, the full area of the map may be shown in size 9 $\frac{2}{3}$ "×14 $\frac{1}{2}$ ", thereby leaving plenty of margin. It may contain about 200 pages, covering about 15 miles of line. It should have durable leather binding.

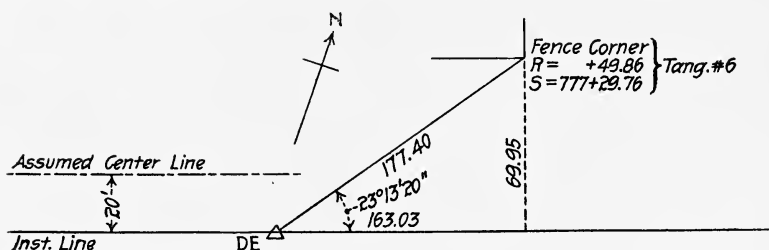
A "map sheet" outline may be shown on every fourth page of the journal corresponding in area (and approximately in number) with the right-of-way maps to be platted, using the reduced scale as pre-

viously mentioned. For instance, sheet No. 1, corresponding to right-of-way map No. 1, may be platted on page 7, sheet 2 on page 11, 3 on 15, etc., leaving some pages in the front of the book for indexing, and the intermediate pages, between the "map sheets," for calculations of surveys near their respective localities, also for calculations of curves connecting tangents, etc.

Calculations.—On the page directly opposite each "map sheet," as, for instance, on page 6 opposite sheet No. 1, sketches of location of all important landmarks, taken from the field notes, should be shown, preferably opposite their true positions on page 7. Calculations of rectangular co-ordinates of these landmarks, referred to the instrument line, assumed line, and adjusted center line, should accompany the sketches if there is room on the page; if not, the results, simply, may be recorded. The former method is far preferable.

Calculations are considered checked only when evidenced by the regular check-mark with the initial of the checker.

If the center line is to be adjusted in the office, the calculations should be made in some neat form with sketch as follows, showing instrument line, assumed center line, and location of landmark:



Range.		Station.	
Log	177.40 = 2.2489536		= 2.2489536
Sin	23-13-20 = 0.5958250		Cos = 0.9633072
69.948	1.8447786	163.027	2.2122608
Assumed.	Adjusted.	Assumed.	Adjusted.
69.948	69.948	DE = 778 + 93.76	778 + 92.79
20.	20.087	1 63.03	1 63.03
+ 49.948	R = + 49.861	777 + 30.73	S = 777 + 29.76

The above shows at a glance the range and station of the fence corner from the instrument line, from the assumed line, and from the adjusted center line, referred to Tangent No. 6. "Assumed line" ranges and stations are used in platting the "adjustment diagram" before the adjusted ranges and stations can be obtained.

Monuments and other landmarks are likewise treated in the same manner.

The assumed ranges of the various points located are very easily determined, of course, by taking into account the instrument-line offset from the assumed. The assumed stationing is obtained by intersecting the assumed center line (by chord connection) with the adjusted center line of the previous tangent, calculating the connecting curve and the positions of its P.C. and P.T. with reference to adjacent instrument points of the survey, and carrying the stationing around the curve from the previously adjusted tangent to the immediate tangent under consideration.

Of course this is all tentative. The adjustment diagram may then be platted, the adjusted center line determined, and from a new calculated intersection with the previous tangent the corrected or adjusted stationing may be projected.

The Adjusted Stationing.—Having determined the adjusted stationing of the first instrument point on the base line (opposite same station on the adjusted center line), as, for instance, point *DA* ($765+52.31$), the stationing of the other points can generally be determined by adding the base-line distances between the points, as follows:

Adjusted Stationing.	Inst. Points.	Check.
$765+52.31$	<i>DA</i>	
$2 \quad 90.91$		
<hr/>		
$768+43.22$	<i>DB</i>	290.91
$2 \quad 96.43$		296.43
<hr/>		291.32
$771+39.65$	<i>DC</i>	298.79
$2 \quad 91.32$		294.13
<hr/>		295.14
$774+30.97$	<i>DD</i>	299.56
$2 \quad 98.79$		<hr/>
<hr/>		2066.28
$777+29.76$	<i>DE</i>	
$2 \quad 94.13$		
<hr/>		
$780+23.89$	<i>DF</i>	$765+52.31$
$2 \quad 95.14$		$20 \quad 66.28$
<hr/>		
$783+19.03$	<i>DG</i>	$786+18.59$
$2 \quad 99.56$		
<hr/>		
$786+18.59$	<i>DH</i>	

A table similar to the above should be made on every map sheet of the journal and thoroughly checked, using the characteristic check-mark and initial of the checker.

From these tables the adjusted stations of the several landmarks are determined in connection with the previously explained calculations, and the results set down adjacent to the monumental points on the calculation sketches and on the map sheets.

It is very desirable to transfer these stations in red ink to the instrument-line points in the field books for ready reference. Of course they are not strictly the stations of these points, but of points opposite on the center line. The angle between the instrument line as run and the adjusted center line will rarely be so great as to require correction for stationing of either line from the other.

MONUMENTING THE ADJUSTED CENTER LINE.

Having projected the adjusted center line on the adjustment diagram and determined its position from the assumed center line by offset distances at base-line points near the ends, its angle with the instrument line can now be obtained and calculations made of the true curve connecting it with the previous tangent, with correct adjusted stationing throughout. The offset distance, therefore, from the instrument line to the adjusted center line can readily be ascertained for any station, and this is essential for center-line monumenting.

Setting the Monuments.—The adjusted center line should be fixed on the ground by stone monuments substantially placed about 500 to 800 feet apart on curves and 1000 to 2000 feet apart on tangents, in order to become readily convenient for use in the field. They may be granite, 6 inches square on top (with top dressed and 1-inch chisel mark down the sides) and about 10 inches square, rough, at the bottom. They should be about $3\frac{1}{2}$ to 4 feet long and set in well-tamped road-bed ballast and earth, mixed. Although concrete backing may be used if desirable, all soft places must be strictly avoided, as near water stations, etc. The top should be at such an elevation that it may readily be found without digging, that is, within view.

In setting monuments it has been found convenient to locate the monument station on the base line, turn a right angle and lay off the required distance to the center line, fixing the point with a tack in stake. This point is then carefully referenced by tacks in ties, using at least one direct lateral and two diagonal measurements.

After the monument has been set the exact center is reproduced from the reference points and marked by a drill-hole $\frac{1}{2}$ inch in diameter and $\frac{1}{2}$ inch deep. The distances from the center of drill-hole to the, say, north and east edges of the monument should be recorded.

Finally, the monument should be referenced from some prominent landmark, such as a street, bridge, or other permanent feature, so it can be readily found in the future.

The Monument Book.—All this information should be recorded in the monument book, which may be an ordinary level book. It should contain the sketches of field operations previously prepared in the office and carefully checked, also references for setting, etc. The date of setting should be recorded without fail; in fact the dates of all field and of most office operations should not be neglected.

THE LEDGER.

As the survey progresses various classes of useful and valuable data will appear and these should all be tabulated in the ledger, being kept up to date abreast of the journal as far as practicable. The journal and ledger may be kept up by the field parties whenever field operations slacken, as during bad weather, etc.

The ledger may be a cross-section book similar to the journal, and special rulings made, as needed, for the various tables of data.

The following tables with column headings are suggested as important:

A. New Alinement as per survey of....(date)....showing also new monuments set on center line of 100 feet right-of-way.....
From....(place)....to....(place)....(main or branch line)
.....R.R.

1. Tangent (or curve) No.
2. Nearest depot.
3. Approximate location (to street line, etc.).
4. Map No. (new).
5. Station (of P.C.'s, P.T.'s, and monuments set).
6. Point (P.C. or P.T.).
7. Degree of curve.
8. Tangent distance (semi-targ.).
9. Intersection angle (*I*).
10. True course of tangent.
11. Monument set (kind).
12. Date of setting.
13. Elevation, U.S.C.S. base.
14. Date, elevation established.
15. Lost (date).
16. Reset (date).
17. Remarks.

Another table, very valuable for permanent record and for making the land maps, is the following:

B. List of Monuments found and located during survey of
(date).... From....(place)....to....(place)....(main or
branch line).....R.R.

1. Monument No.
2. Tangent (or curve) No.
3. Nearest depot.
4. Approximate location (to street line, etc.).
5. Map No. (new).
6. { Stations and } Station.
7. { Ranges from } Range (north).
8. { Established } Range (near center).
9. { Center line } Range (south).
10. Kind of monument.
11. Set by whom.
12. Date of setting (*a*=antedates).
13. Source of information.
14. Elevation, U.S.C.S. base.
15. Date, elevation established.
16. Street monuments.
17. Condition of monument.
18. { Correction } Station, + or -.
19. { if righted } Range, + or -.
20. When located.
21. When lost or destroyed.
22. Remarks.

For Column No. 1, the monument number should indicate the tangent on which it is situated as well as its own relative position on the tangent, as, for instance, monuments on Tangent No. 1 should be numbered consecutively 101, 102, 103, etc., those on Tangent No. 8, 801, 802, 803, etc. When tables are practically complete, if a monument should subsequently be found and located, the position of which on the ground would place it between, say, 623 and 624, it can be numbered 623*a*, and so on.

If a monument were adjacent to a curve, its range and station would be calculated with reference to one or both of the tangents produced, and perhaps also from the center line of the curve. Suppose, for illustration, monument No. 735 to be calculated with reference to Tangent No. 7, produced beyond the P.C. of the curve connecting

it with Tangent No. 8. If it were also calculated with reference to Tangent No. 8, its number would remain 735, but its tangent number, in Column No. 2, would be 8 instead of 7. If it be calculated from the center line of curve also, its tangent number, 735, would still be preserved for Column No. 1, while in Column No. 2 would appear "Curve 7-8" or "Cv. 7-8," indicating that its range and station are referred to the curve joining Tangents 7 and 8. Hence a monument situated on a curve may require three lines on the ledger sheet, one for each tangent and one for the curve itself.

In this connection it will be well to state that from the ledger sheet the kind of monument and its number may be recorded in the journal, adjacent to the monumental point of the "calculation sketch" and the "map sheet," in blue ink on the former and in pencil on the latter, thus:

- | | | |
|---|---|--|
| (1) $R = +364.19$
$S = 1531 + 81.62$ | } | Stone M, No. 877
Tang. No. 8 + (Ref. to Tang. No. 8 produced beyond P.C.). |
| (2) $R = -50.32$
$S = 1532 + 11.07$ | } | Stone M, No. 877
Cv. Nos. 8-9 (Ref. to curve con. Tang. No. 8 and No. 9). |
| (3) $R = +298.01$
$S = 1532 + 31.26$ | } | Stone M, No. 877
Tang. No. 9 - (Ref. to Tang. No. 9 produced back of P.T.). |

Note.—In the case of (1) and (3) the range is +, or north, while for (2) it is -, or south.

For "kind of monument," Column No. 10, abbreviations may be used as, for instance, "C.I.M." for cast-iron monument, "I. Bar" for iron bar, "S.M." or "Stone" for stone monument, etc.

The "date of setting," Column No. 12, may be obtained from office or city records, or perhaps from the local residents. When not absolutely sure of the date a question mark should accompany it. In the case of a monument being referred to in deeds or other authentic records as having existed at a certain date the letter (*a*) may be placed before it to indicate that the monument was set prior to that time, as (*a*) June, 1867, means "antedates June, 1867."

For Column No. 16, street monuments, the name of the street or streets should be given, and in the column under "Remarks" the description of the precise location with reference to the centers, side lines, or corners of the streets should be noted.

The condition of the monument, Column No. 17, is quite important to record. It may be loose, leaning, top broken off, good or other-

wise. If it is leaning, as indicating a change of position, the dip with the perpendicular will be obtained in the field and correction calculated for "righting" as per Columns Nos. 18 and 19. The correction, + or -, must be used algebraically with its recorded range and station to get its presumed original position. The exception to this rule will be where the range and station of the original position or presumed original position of the monument is recorded direct, in which case some note, as "Original position," should accompany it in the "Remarks" column.

Column No. 21, "when lost or destroyed," is for future record.

The structural sheet, made up from the right-of-way survey, will be invaluable as an aid to mapping and side-monumenting. Its characteristic feature is the record of encroachments.

C. List of Structures (fences, buildings, bridges, retaining walls, etc.) located during survey of....(date).....

From....(place)....to....(place).... (main or branch line)
.....R.R.

1. Tangent (or curve) No.
2. Nearest depot.
3. Approximate location.
4. Map No. (new).
5. { Stations and } Station.
6. { Ranges from } Range (north).
7. { Established } Range (near center).
8. { Center line } Range (south).
9. Deed range.
10. { Encroachment } On.....R.R.
11. { } By.....R.R.
12. Kind of structure.
13. Point located.
14. Built by whom.
15. When located.
16. Condition.
17. When destroyed or removed.
18. When replaced.
19. Remarks.

Note.—In Column No. 9, "deed range," set down the distance from the established center line to the intended position of the object as interpreted from the deeds. Distances in Columns Nos. 10 and 11 are simply the difference between the actual range of the structure and the intended or deed range.

SIDE-MONUMENTING.

GENERAL DISCUSSION.

SIDE-MONUMENTING should be done in connection with the mapping with extreme care and with legal advice. Much valuable time will be lost and needless expense incurred if the side-monumenting is delayed until the land maps are otherwise completed. Of course the finished maps will include the side monuments as set.

After a side monument has been set it would be extremely difficult for the company to claim beyond it by moving "out," and humiliating to retreat; hence the utmost care must be exercised in setting them.

Old records should be carefully examined, especially for any dispute that may have arisen in regard to the boundary line; and if such has come up in the past, it might be wise to call in the local surveyor to adjust the line and set the monuments.

Information for setting the side monuments should be worked up in the office and taken into the field. There, however, the practical conditions on the ground may be such as to slightly change or completely alter the office data for laying out. In other words, it may be necessary to be governed more or less by conditions on the ground rather than to adhere strictly to the office notes, although the latter will be at least a valuable aid.

Where to Side-monument.—Monuments should be set as follows:

(a) On all important exterior jogs, corners, and bends of the right of way.

(b) Where (a) is impracticable owing to existing obstructions, a reference monument may be set in as serviceable a position as possible, preferably on the right-of-way line (sometimes on the land line) and at a certain recorded distance from the true corner or bend, far enough away so it cannot possibly be mistaken for the latter.

(c) On the right-of-way lines opposite the P.C. and P.T. of curves.

(d) At intersection of important land lines with the right of way, even if there are no jogs in the latter.

(e) Frequently along right-of-way lines, say opposite some of the center-line monuments if other side monuments are not close at hand. This includes curves as well as tangents.

(f) Wherever there is reason to fear encroachment or where it already exists.

A table may be prepared with the following headings to meet ordinary conditions for side-monumenting and for use in platting the land maps. Part of the data are to be worked up from office notes and plans, and from the ledger sheets and land maps as they progress.

D. Side-monument Sheet showing monuments set (and to be set?) at critical points on the right-of-way boundary and property lines. Survey of....(date)....

From.....(place).....to.....(place)....(main or branch line)
.....R.R.

1. Tangent (or curve) No.
2. Nearest depot.
3. Approximate location.
4. Map No. (new).
5. { Stations and } Station.
6. { Ranges from } Range (north).
7. { Established } Range (near center).
8. { Center line } Range (south).
9. Deed range from new center line.
10. Angle between land line and new center line.
11. New station at intersection of same.
12. Kind of line.
13. Monument set.
14. Date of setting.
15. When lost or destroyed.
16. Old angle of land line with old center line.
17. { Old stationing } Crossing old center line.
18. { } Crossing new center line.
19. Reference to old notes.
20. Deed range from old center line.
21. Remarks.

CITY AND TOWN SURVEYS.

As the right-of-way survey progresses, more or less elaborate city and town work is required. It is a good plan to locate every monument in sight, wherever found. If the monuments located do not define the position of the streets, the street lines should be run out and connected with the center line of the railroad, using offsets to buildings if they can be obtained from the local surveyors. As a last resort street fences and curbs may be used to fix the lines.

The amount of city work to be done will depend upon the company's present and prospective interests in the locality. All street lines crossing the right of way as well as those parallel with and a short distance from it should be accurately determined.

Special care should be exercised in determining old as well as new street lines and when the changes, if any, were made. This matter becomes quite important when it comes to platting deeds referring to "street lines" on the maps.

Of course the ranges and stations of all monuments located are to be calculated in the proper manner and place in the journal and the results recorded in the ledger, sheet B.

CONNECTING WITH GOVERNMENT STATIONS.

FROM the table of geographical positions of church spires, chimneys, flagstaffs, etc., which have been determined with accuracy by the government and state surveys, many points may be selected along the line of the survey and located, generally by triangulation.

The position, then, of any point on the survey can be determined by latitude and longitude, and also the true bearings of the tangents. The latter is very important in relation to the magnetic bearings of the old land lines where the dates of same are known.

The latitudes and longitudes should be reduced to Clarke's spheroid and to the nearest hundredth of a second.

LEVELING.

IF it is important to establish a monumented alinement on the ground as a backbone for future surveys, it is likewise desirable to establish a system of bench-marks along the line for convenient and consistent leveling. A good base to adopt is that of mean sea-level as established by the U. S. Coast and Geodetic Survey. Equated levels may be obtained with the several city and town bases along the line of the road.

ESTABLISHING BENCH-MARKS.

In advance of the leveling, permanent benches should be established in firm positions on masonry structures every half mile if possible. A $\frac{3}{4}$ " copper rivet with a boiler-rivet head (flat at the top) set into a hole in the masonry either by driving or with cement makes a very desirable and lasting bench-mark. If provided with a kerf at the lower end and a steel wedge for spreading it when driven, additional security will result.

A good formula for accurate leveling of this character is the following:

Error, up and back, not to exceed $.016 \text{ ft.} \sqrt{\text{distance in miles, one way.}}$

The following system of recording permanent and temporary bench-marks, established, is recommended for the ledger:

E. List of Permanent and Temporary Bench-marks established during survey of . . . (date) . . . From . . . (place) . . . to . . . (place) . . . (main or branch line) . . . R.R.

1. Tangent (or curve) No.
2. Nearest depot.
3. Approximate location.
4. Permanent B.M. No. (using Roman numerals, as I, II, etc.).
5. Temporary B.M. No. (using Arabic numerals, as 1, 2, etc.).
6. Kind of B.M.

7. Station (new).
8. Distance from center line, N. or S
9. Description (accurate).
10. Elevation in feet, U.S.C.S. base.
11. Date, elevation established.
12. When destroyed.
13. Elevation, south rail, Track No. (opposite).
14. Reference, Level Book.
15. Remarks.

Note.—In the column of “Remarks” may be noted the equated elevations of bench-marks with city bases.

ELEVATIONS OF MONUMENTS.

From the established bench-marks the elevations of all monuments should be obtained and recorded in the columns of the ledger sheets as described for that purpose.

ELEVATIONS OF TRACK AND BRIDGES.

The elevation of the highest rail of each track and the clear head-room for same at each overhead bridge or other structure should be ascertained, and recorded on the structural maps.

PHYSICAL FEATURES.

In connection with the structural maps, which will be described later, a few of the main physical features may well be shown. The economy of having this information in the office might well be illustrated in the case of a contemplated siding. If the structural maps are made to show the positions of excavations and embankments, general ground elevations, etc., the superintendent can generally determine at a glance whether the siding would be practicable, and the engineer can render an estimate, quickly, without field expense.

Side Slopes.—The range, station, and elevation of points of change in top and bottom of slopes should be taken, and the character of the material noted, as far as practicable.

Retaining Walls.—Range, station, and elevation of top front and bottom front of wall, depth of foundation (from office records, perhaps), and character of wall should be noted.



Side Levels.—These may be taken to the nearest tenth of a foot, to the limit of the right of way, or, in some instances, a little beyond. The ordinary judgment of cross-sectioning will determine the frequency and amount of leveling to be done.

THE MAPPING.

THE mapping should begin immediately with the adjustment of the first tangent, be carried on uninterruptedly, and as it is by far the slowest and most tedious operation it should be taken up at various advanced points along the line as the field work progresses.

THE SCALE.

Selecting the scale for the maps is an important consideration. It is largely dependent upon the requirements which the maps have to meet; the nature, extent, and system of existing records; also the accuracy of proposed survey. The latter will be assumed to be in accordance with the methods herein described.

The usual scale is 100, and this may be used for outlying sections, simple in character, and where company property extends considerably beyond the right of way. It will be found, however, that a scale of 50 is none too large for maps embracing strictly railroad properties in populated districts, with many and complicated land lines, monuments, etc., to register. With some roads which have acquired their right of way in piecemeal, as, for instance, 25- or 33-ft. strips, widened to 50 or 66 feet, and again later to 100, with one or more old center lines, it would be necessary to use a 50-ft. scale.

In the case of terminals, etc., where both conditions exist, it is sometimes desirable to use both scales, making two sets of maps.

LAND AND STRUCTURAL MAPS COMPARED.

The land and the structural maps should, generally, be separate, but have similar numbers and embrace similar areas.

The land maps may include such structural features as will explain more clearly certain agreements, as, for instance, the allowed encroachments of abutments or other structures within the right of way, etc.

On the other hand, the structural maps should show, in addition to the structural and physical features, the exact position of the company's exterior boundary lines, accurately defined so they can be produced on the ground.

THE LAND MAPS.

The Paper.—The original land maps should be on durable mounted white paper of close texture and character which will stand erasing, and not fade too much under ordinary light. The best is none too good.

When finished to date they may be bound in book form, or remain as loose sheets and filed in shallow, removable, sheet-steel or light wooden drawers with "drop sides."

They should be kept up to date from records filed with the Real Estate Department.

A set of tracings may be made for the Law Department and kept up to date in a similar manner, but promiscuous blue-printing should be avoided.

Laying out the Map.—Having selected the scale, say 50, and the size of the map, say 28"×42" inside of border (sheet 29"×43"), bisect the sheet longitudinally by a base line and lay off the border line very accurately. This base line becomes, generally, the adjusted center line of the tangent or tangent produced, as the case may be. It should be drawn in red (carmine) ink and the stationing laid off, with red circles every fifth station, at the "5" and "10" points, unless a stone monument has been set there, in which case it would have the "stone monument" symbol, a square outline in black.

It is well to project "500-ft." range and station lines *outside the border*, thus blocking the map off in 500-ft. squares, virtually preserving the scale. The 500-ft. stationing may also be noted on these lines at the top of the sheet.

Platting.—From the journal, ledger, abstracts of deeds, old plans, etc., the maps are constructed. The field-survey notes, worked up in the journal and ledger, will show the true positions of all streets and street monuments in the vicinity of the railroad, all important structures which may have a bearing in determining the positions of land lines, and any monuments previously set along the right of way.

With these practical data which have been reduced to rectangular coordinates for simplified position the deeds may readily be located and platted on the maps.

The platting should be as accurate as possible and not a mere picture map with wide shaded street lines, etc. Lines should be shown firm and clear but not heavy. It is a good rule to show the present

right-of-way lines, or rather outer boundaries, in full, and the numerous interior lines due to piecemeal purchases, etc., dotted. If some of the latter are more important than others, heavier or longer dots may be used.

Vacated streets should be shown dotted, but perhaps the center line of the street in full if it form an important land line. In cases of abandoned streets the original abutting owners may justly claim to the center line (with right-angle side lines) by reversion. It is very difficult in many cases to interpret both legally and practically, but recourse can always be had to the Law Department, and in fact the whole work should be done in very close touch therewith, as many legal questions arise in settling land lines, side-monumenting, etc.

Old and New Center Lines.—If there be an old center line, its position will be revealed by the platting. It may or may not be identical with the recently adjusted center line.

If the two are identical, the new stationing may be on one side of the line, say the upper, and the old stationing on the other, the lower. On the lower side of the line, then, reference will be made to the old center line, giving date of filing, magnetic bearing, etc. On the upper side of the line will be recorded notes regarding the adjusted center line, with date of same, also the "true bearing" to the nearest one quarter minute. The true bearing may be obtained by solar work or from the survey connecting the government and state triangulation stations.

If the old center line be not identical with the new, it can be shown in vermilion ink as distinguishing it from the adjusted (in carmine), and each designated line titled separately. If it were, say, the center of a former 66-ft. right of way and subsequently an additional strip (34 feet) were purchased on one side, making the present right of way 100 feet, the two center lines would be shown.

Another case to be met with is where the old filed center line was found to be sinuous in years back, and an attempt was made to straighten it at the time of acquiring additional right of way. The present re-adjustment will, of course, attempt to follow closely the "straightened" line even if a "bend" is found necessary in the "tangents."

Subsequent to the "straightening" some of the deeds, perhaps intentionally, may refer to it as the filed line. This will doubtless cause some confusion in addition to that arising from other quarters, such as deeds giving wrong bearings, scaled distances using the wrong scale, etc. These problems can usually be solved with care.

Symbols.—Recognized symbols may be used as follows:

- = iron monument;
- with the particular name after it may be used to designate any kind of monument.
- = stone monument.

The ranges and stations of all monuments should be shown with reference to the adjusted tangent if they are on the tangent. If the monument is adjacent to a curve, its range and station will be shown with reference to the curve and to one or both of the tangents produced.

For illustration,

$$\begin{array}{c} \text{C.I.M.} \\ \circ \left\{ \begin{array}{l} R = +49.96 \\ S = 123 + 13.44 \end{array} \right\} \text{Tang. No. 8+} \end{array}$$

means, "Cast-iron monument, range north 49.96 from station 123 + 13.44 referred to Tangent No. 8 produced toward No. 9." Likewise, "Tang. No. 8—," after the bracket, would mean, "Tangent No. 8, produced toward No. 7," while the absence of the algebraic sign altogether would signify, simply, "Tangent No. 8," itself. The plus sign before the range indicates "north," and the minus sign "south."

Again,

$$\square \left\{ \begin{array}{l} R = -49.96 \\ S = 123 + 13.44 \end{array} \right\} \text{Cv. Nos. 8-9}$$

means, "Stone monument, range south 49.96 from station 123 + 13.44 referred to curve connecting Tangents Nos. 8 and 9."

Generally the absence of "Cv..." after the bracket will signify that the range and station are referred to the tangent or tangent produced.

Index to Transfers.—The various transfers are shown on the lots by numbers in red ink inside of red circles, or if the lot is very small the circle may be omitted.

An index, or list of transfers, is shown on each land map in a permanently vacant space where it will not interfere with the map proper, and each transfer is recorded under the following headings:

1. Office File (as "Essex 36"—Essex County, deed No. 36).
("Essex 36" in red for purchases and green for sales.)
2. Grantor.
3. Grantee.
4. Map No. (of lot), as 136*a*, 136*b*, etc.

5. Date (of deed).
6. Conveyance (as W=warranty, Q.C.=quitclaim, etc.).
7. Record Book (in office).
8. Page (of Record Book).
9. Remarks (special clauses and agreements).

Property Lines.—A narrow thin liquid carmine shading along the outline of the company property shows clearly present ownership, while a liquid green shading shows property formerly owned by the company. As transfers take place the new coloring can be put on, even over the old, and the maps kept up to date.

Some theoretical objection may be made to this method, but it can be truly stated that the new coloring shows quite clearly, and that maps are in existence which have been kept up for thirty-five years in this manner, through periods of numerous land transfers, both buying and selling, and clearly show the properties. They are the main record to-day.

The conventional —·—·—· may be used if preferred. Much will depend upon local conditions.

North Point.—The north point should be in some obscure part of the sheet free from interference with present or future matter. It should be the “true” north and so marked.

Explanatory Notes.—Near the north point in all cases should be the special or explanatory notes for stationing, monuments, coloring, true bearing, etc.

Title.—The title of the map should be outside the border line, part above the upper and part below the lower, along the $\frac{1}{2}$ -inch margins at the right hand of the sheet. For instance, at the top of the map may be the name of the road, main line or branch and sheet number, while at the bottom may be the general title of the adjustment survey, scale, date, and sheet number repeated.

A Base for the Structural Maps.—Besides serving their regular purpose in the land office these maps constitute a base for the structural maps, which should closely follow and which will now be explained.

THE STRUCTURAL MAPS.

The structural maps will embrace considerable information from the land maps, together with the structural and certain physical features along the line.

From the Land Maps.—In direct contrast with the land maps, the structural maps should be on tracing cloth for the purpose of furnishing blue-prints when required. Generally they should be to the same scale as the land maps, and from the latter may be traced directly the adjusted center line, center-line monuments, side lines, side-line monuments, intersecting land lines, streets, etc. They need show only the "outline" of present ownership and not the "built-up" right of way as it appears on the land maps.

The side lines may be correctly exhibited by showing the positions of "points of bends" by ranges and stations from the center line, also the angles at these bends and the lengths of side lines, where advisable.

The alinement will be that of the adjusted center line as monumented.

Company property lines may be shown on the tracings by the standard dash, dot, dot, dash, dot, dot, etc., somewhat heavier than the dash, dot, dash, dot used to represent fences. A fine red crayon shading may also be carried around inside of these lines if desirable to bring out more clearly the company property.

Structural Features.—On these maps should be shown all tracks, bridges, abutments, culverts, buildings, fences, signals, etc., in fact all structural features. They must be drawn carefully to scale, and furthermore the ranges and stations recorded for all important or critical points, as fence or building corners, frog-points, etc. In cases of long stretches of track or side fences, apparently straight, the ranges *only* need be shown opposite "round stations" on the center line.

Economic Method.—It would be a waste of time, involving needless expense, to start out in the field and measure up *everything* regardless of hundreds of existing surveys and plans in the office of the company which can be utilized by "tying them in" carefully to the monumented center line. Although not strictly modern, they will, many of them, answer the purpose temporarily, to be supplanted later with information of a more definite character during the ordinary course of work, without added expense. Judgment must be used as to whether entirely new surveys are essential in certain localities.

Maps compiled from office records, or from new surveys, should so be designated.

Whenever new surveys are made or changes occur in location of track or structures of any kind, tracing plans of such changes will be made. These should become identified with the structural maps of the particular localities and blue-prints of same, or at least memo-

randa, filed therewith. In this way small changes and additional information may be kept track of, and after a certain length of time, or when a sufficient number of changes have taken place, new structural tracings may be made embracing all changes to date. Blue-prints of these tracings should be sent out to supersede all previous prints furnished, which will be filed away as matter of record or history.

Physical Features.—Contour lines cannot well be shown, but the edges of slopes in excavation and embankment may be outlined, with elevations along same at intervals, and also beyond to limit of right of way.

Elevations of top of rail, top and bottom of retaining walls, bridge seats, depot platforms, etc., will suggest themselves as important data for the maps. The clear head-room at each crossing referred to actual elevations from the adopted base will be most convenient.

THE INDEX MAPS.

For ready reference to the land and structural maps the index maps are prepared, platted on a true polyconic projection, using Clarke's spheroid.

Adopting a scale of 4000 feet to the inch, the parallels of latitude may be shown every two minutes and the meridian lines $2\frac{1}{2}$ minutes apart.

The government triangulation stations, which have been located by the survey with a view to adjusting the alinement on the maps, are platted by latitude and longitude.

The Alinement.—The alinement and triangulation stations are platted on tracing cloth by the survey notes, matched on the index map by these stations, and the alinement produced thereon in its proper geographical position.

Along the center line as platted are shown the stationing, every 1000 feet, the names of depots, and the outline position of each land or structural map to scale, with map number of same in red.

True Bearings.—The true bearings of tangents may be obtained in the following manner:

a. Calculate the true bearing of the imaginary line joining two important government stations, situated some distance apart and which have been located by the survey, using the latitudes and longitudes of the stations, reducing the differences to feet, and solving the triangle as a right-angle triangle. The angle obtained will give the

true divergence of the line from the meridian passing through its middle point.

b. Project this meridian on the map and note, by scale, the station where it intersects the center line.

c. The angles worked up from the survey traverse, in connection with the true bearing of the line joining the government stations, as above, will point to the true bearing of the tangent at its station of intersection with said meridian.

As meridians converge toward the north, it is evident that for any tangent running easterly and westerly its true bearing will vary at different points on the tangent. For instance, on an east-and-west line in the latitude of New York there is a variation of about one second in bearing at points 117 feet apart, or one minute in about $1\frac{1}{3}$ miles. The distance apart of points on any line in this latitude for a variation in bearing of one second will consequently be $117 \times \text{cosecant of true bearing angle of line with the meridian}$.

In this manner, considering the average latitude or calculating for different average latitudes, as the case may demand, the true bearings may be obtained to the second for any tangent at any point. They should be calculated at every stone monument and recorded in Column No. 10 of the alinement sheet.

APPENDIX.

HINTS FOR FIELD WORK.

TABLE OF TEMPERATURE CORRECTIONS.

TABLE OF ALLOWABLE ERRORS IN LEVELING.

TABLE OF NATURAL TANGENTS AND COTANGENTS FOR PLATTING.

TABLE OF FEET AND CHAINS.

TRIGONOMETRIC FORMULAS.

APPENDIX.

HINTS FOR FIELD WORK.

It might not be out of place here to speak of some apparently simple matters (but in reality far-reaching) in connection with work on the line.

As men are greatly separated at times, or ordered beyond speaking distance (this in the truly literal sense) to perform certain harmonious duties, they must keep wide awake. Codes of signals will be adopted, but they should be simple and few. Men will learn to understand each other as they become drilled to the work.

Cheap men are not economical. They are an encumbrance and liable by serious errors to render the work not only expensive but questionable.

The assistant in charge of the field party should take the notes, keep his eyes open, and not be obliged to run the instrument.

In chaining, the hind chainman should hold the zero end of the tape constantly on the point while the head chainman is taking the reading. As the tape cannot be held absolutely fixed in position continuously, the hind chainman should, when he thinks the head chainman is ready, call sharply "right," "right," at frequent intervals. The head chain takes the reading or readings at such instants, and if they are satisfactory he answers "all right." If the hind chain is slow, careless, and awkward, many errors will result. A good custom is for the head chainman to read the foot-mark beyond the true measurement, as well as the one before it, as a check.

Constant attention to the work at hand will secure not only rapid but accurate results, and they cannot be secured otherwise.

TABLE OF TEMPERATURE CORRECTIONS.

Assuming the coefficient of expansion for tape-steel to be .000006 $\frac{2}{3}$ per degree Fahrenheit, there would be a change of .01 feet in a 100-foot tape for each 15 degrees variation in temperature.

The following table shows corrections in decimals of a foot per 100 feet (for tapes standard at 40° to 85°) to be applied to measurements taken at temperatures varying from 0° to 100° F. For instance, if the tape be standard at 60° and the measurement taken at 20°, the correct measurement would be .027 foot less per 100 feet.

TABLE.

		100-foot Tape Standard at following Temperatures.										
Field Temperatures.		40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	
	0°	—	.027	.033	.037	.04	.043	.047	.05	.053	.057	0°
	5°	—	.023	.027	.03	.037	.04	.043	.047	.05	.053	5°
	10°	—	.02	.023	.027	.03	.033	.037	.04	.043	.047	10°
	15°	—	.017	.02	.023	.027	.03	.033	.037	.04	.043	15°
	20°	—	.013	.017	.02	.023	.027	.03	.033	.037	.04	20°
	25°	—	.01	.013	.017	.02	.023	.027	.03	.033	.037	25°
	30°	—	.007	.01	.013	.017	.02	.023	.027	.03	.033	30°
	35°	—	.003	.007	.01	.013	.017	.02	.023	.027	.03	35°
	40°	—	—	.003	.007	.01	.013	.017	.02	.023	.027	40°
	45°	+	.003	—	.003	.007	.01	.013	.017	.02	.023	45°
	50°	+	.007	+	.003	—	.003	.007	.01	.013	.017	50°
	55°	+	.01	+	.007	+	.003	—	.003	.007	.01	55°
	60°	+	.013	+	.01	+	.007	+	.003	—	.003	60°
	65°	+	.017	+	.013	+	.01	+	.007	+	.003	65°
	70°	+	.02	+	.017	+	.013	+	.01	+	.007	70°
	75°	+	.023	+	.02	+	.017	+	.013	+	.01	75°
	80°	+	.027	+	.023	+	.02	+	.017	+	.013	80°
	85°	+	.03	+	.027	+	.023	+	.02	+	.017	85°
	90°	+	.033	+	.03	+	.027	+	.023	+	.02	90°
	95°	+	.037	+	.033	+	.03	+	.027	+	.023	95°
	100°	+	.04	+	.037	+	.033	+	.03	+	.027	100°

The above signs, + and —, are to be adhered to in making temperature corrections for measurements between *fixed* objects in the field. That is, if the actual temperature when the measurement is taken is *below* the tape standard temperature, the correct distance is less than the tape reading and hence the correction is *minus*. For an actual temperature *above* that of the tape standard the correction is *plus*.

For *laying out certain* distances (as in staking out monuments) on the ground, the *apparently reverse* operation holds true. If the actual temperature is *below* that of the tape standard, the temperature correction should be *added*, and if *above*, *subtracted*.

TABLE OF ALLOWABLE ERRORS IN LEVELING.

Distance, Miles.	Distance, Feet.	Allowable Error.*	Distance, Miles.	Distance, Feet.	Allowable Error.*
.1	528	.005	.0189	100	.0022
.125	660	.006	.04	200	.003
.25	1320	.008	.06	300	.004
.333	1760	.009	.08	400	.004
.5	2640	.011	.09	500	.005
.625	3300	.013	.11	600	.005
.75	3960	.014	.13	700	.006
1	5280	.016	.15	800	.006
1.25	6600	.018	.17	900	.007
1.5	7920	.020	.19	1000	.007
1.75	9240	.021	.23	1200	.008
2	10560	.023	.27	1400	.008
2.5	13200	.025	.30	1600	.009
3	15840	.028	.38	2000	.010
4	21120	.032	.47	2500	.011
5	26400	.036	.57	3000	.012
6	31680	.039	.66	3500	.013
8	42240	.045	.76	4000	.014
10	52800	.051	.85	4500	.015
12	63360	.055	.95	5000	.016
15	79200	.062	1.14	6000	.017
20	105600	.072	1.33	7000	.018
25	132000	.080	1.52	8000	.020
30	158400	.088	1.70	9000	.021
40	211200	.101	1.89	10000	.022
50	264000	.113	2.08	11000	.023
60	316800	.124	2.27	12000	.024
70	369600	.134	2.46	13000	.025
80	422400	.143	2.65	14000	.026
90	475200	.152	2.84	15000	.027
100	528000	.160	3.03	16000	.028
125	660000	.179	3.22	17000	.029
150	792000	.196	3.41	18000	.030
175	924000	.212	3.79	20000	.031
200	1056000	.226	4.17	22000	.033
250	1320000	.253	4.73	25000	.035
300	1584000	.277	5.68	30000	.038
350	1848000	.299	6.63	35000	.041
400	2112000	.320	7.58	40000	.044
450	2376000	.339	8.52	45000	.047
500	2640000	.358	9.47	50000	.049

* Error (in feet) leveling "up and back" must not exceed
 $.016\sqrt{\text{distance in miles (one way)}}$.

TABLE OF NATURAL TANGENTS AND

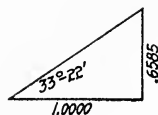
Diff. for 1'.	Tan- gents.	0'	5'	10'	15'	20'	25'	30'	
2.9	0°	.0000	.0015	.0029	.0044	.0058	.0073	.0087	80°
2.9	1°	.0175	.0189	.0204	.0218	.0233	.0247	.0262	88°
2.9	2°	.0349	.0364	.0378	.0393	.0407	.0422	.0437	83°
2.9	3°	.0524	.0539	.0553	.0568	.0582	.0597	.0612	86°
2.9	4°	.0690	.0714	.0729	.0743	.0758	.0772	.0787	85°
2.9	5°	.0875	.0890	.0904	.0919	.0934	.0948	.0963	84°
2.9	6°	.1051	.1066	.1080	.1095	.1110	.1125	.1139	83°
3.0	7°	.1228	.1243	.1257	.1272	.1287	.1302	.1317	82°
3.0	8°	.1405	.1420	.1435	.1450	.1465	.1480	.1495	81°
3.0	9°	.1584	.1599	.1614	.1629	.1644	.1658	.1673	80°
3.0	10°	.1763	.1778	.1793	.1808	.1823	.1838	.1853	79°
3.0	11°	.1944	.1959	.1974	.1989	.2004	.2019	.2035	78°
3.0	12°	.2126	.2141	.2156	.2171	.2186	.2202	.2217	77°
3.1	13°	.2309	.2324	.2339	.2355	.2370	.2385	.2401	76°
3.1	14°	.2493	.2509	.2524	.2540	.2555	.2571	.2586	75°
3.1	15°	.2679	.2695	.2711	.2726	.2742	.2758	.2773	74°
3.2	16°	.2867	.2883	.2899	.2915	.2931	.2946	.2962	73°
3.2	17°	.3057	.3073	.3089	.3105	.3121	.3137	.3153	72°
3.2	18°	.3249	.3265	.3281	.3298	.3314	.3330	.3346	71°
3.3	19°	.3443	.3460	.3476	.3492	.3508	.3525	.3541	70°
3.3	20°	.3640	.3656	.3673	.3689	.3706	.3722	.3739	69°
3.4	21°	.3839	.3855	.3872	.3889	.3906	.3922	.3939	68°
3.4	22°	.4040	.4057	.4074	.4091	.4108	.4125	.4142	67°
3.4	23°	.4245	.4262	.4279	.4296	.4314	.4331	.4348	66°
3.5	24°	.4452	.4470	.4487	.4505	.4522	.4540	.4557	65°
3.6	25°	.4663	.4681	.4699	.4716	.4734	.4752	.4770	64°
3.6	26°	.4877	.4895	.4913	.4931	.4950	.4968	.4986	63°
3.7	27°	.5095	.5114	.5132	.5150	.5169	.5187	.5206	62°
3.8	28°	.5317	.5336	.5354	.5373	.5392	.5411	.5430	61°
3.8	29°	.5543	.5562	.5581	.5600	.5619	.5639	.5658	60°
3.9	30°	.5774	.5793	.5812	.5832	.5851	.5871	.5890	59°
4.0	31°	.6000	.6028	.6048	.6068	.6088	.6108	.6128	58°
4.1	32°	.6249	.6269	.6289	.6310	.6330	.6350	.6371	57°
4.2	33°	.6494	.6515	.6536	.6556	.6577	.6598	.6619	56°
4.3	34°	.6745	.6766	.6787	.6809	.6830	.6851	.6873	55°
4.4	35°	.7002	.7024	.7046	.7067	.7089	.7111	.7133	54°
4.5	36°	.7265	.7288	.7310	.7332	.7355	.7377	.7400	53°
4.6	37°	.7536	.7558	.7581	.7604	.7627	.7650	.7673	52°
4.7	38°	.7813	.7836	.7860	.7883	.7907	.7931	.7954	51°
4.9	39°	.8008	.8122	.8146	.8170	.8195	.8219	.8243	50°
5.0	40°	.8391	.8416	.8441	.8466	.8491	.8516	.8541	49°
5.1	41°	.8693	.8718	.8744	.8770	.8796	.8821	.8847	48°
5.3	42°	.9004	.9030	.9057	.9083	.9110	.9137	.9163	47°
5.5	43°	.9325	.9352	.9380	.9407	.9435	.9462	.9490	46°
5.7	44°	.9657	.9685	.9713	.9742	.9770	.9798	.9827	45°
		60'	55'	50'	45'	40'	35'	30'	Cotan- gents.

Example.—To plat angle 33° 22':

$$\tan 33^\circ 20' = .6577$$

$$\text{d. } 4.2 \times 2 = 8$$

$$\tan 33^\circ 22' = .6585$$



COTANGENTS FOR PLATTING.

Tan- gents.	35'	40'	45'	50'	55'	60'		Diff. for 1'.
0°	.0102	.0116	.0131	.0145	.0160	.0175	80°	2.9
1°	.0276	.0291	.0306	.0320	.0335	.0349	88°	2.9
2°	.0451	.0466	.0480	.0495	.0509	.0524	87°	2.9
3°	.0626	.0641	.0655	.0670	.0685	.0699	86°	2.9
4°	.0802	.0816	.0831	.0846	.0860	.0875	85°	2.9
5°	.0978	.0992	.1007	.1022	.1036	.1051	84°	2.9
6°	.1154	.1169	.1184	.1198	.1213	.1228	83°	2.9
7°	.1331	.1346	.1361	.1376	.1391	.1405	82°	3.0
8°	.1509	.1524	.1539	.1554	.1569	.1584	81°	3.0
9°	.1688	.1703	.1718	.1733	.1748	.1763	80°	3.0
10°	.1868	.1883	.1899	.1914	.1929	.1944	79°	3.0
11°	.2050	.2065	.2080	.2095	.2110	.2126	78°	3.0
12°	.2232	.2247	.2263	.2278	.2293	.2309	77°	3.1
13°	.2416	.2432	.2447	.2462	.2478	.2493	76°	3.1
14°	.2602	.2617	.2633	.2648	.2664	.2679	75°	3.1
15°	.2789	.2805	.2820	.2836	.2852	.2867	74°	3.1
16°	.2978	.2994	.3010	.3026	.3041	.3157	73°	3.2
17°	.3169	.3185	.3201	.3217	.3233	.3249	72°	3.2
18°	.3362	.3378	.3395	.3411	.3427	.3443	71°	3.2
19°	.3558	.3574	.3590	.3607	.3623	.3640	70°	3.3
20°	.3755	.3772	.3789	.3805	.3822	.3839	60°	3.3
21°	.3956	.3973	.3990	.4006	.4023	.4040	68°	3.4
22°	.4159	.4176	.4193	.4210	.4228	.4245	67°	3.4
23°	.4365	.4383	.4400	.4417	.4435	.4452	66°	3.5
24°	.4575	.4592	.4610	.4628	.4645	.4663	65°	3.5
25°	.4788	.4806	.4823	.4841	.4859	.4877	64°	3.6
26°	.5004	.5022	.5040	.5059	.5077	.5095	63°	3.7
27°	.5224	.5243	.5261	.5280	.5298	.5317	62°	3.7
28°	.5448	.5467	.5486	.5505	.5524	.5543	61°	3.8
29°	.5677	.5696	.5715	.5735	.5754	.5774	60°	3.9
30°	.5910	.5930	.5949	.5969	.5989	.6009	59°	3.9
31°	.6148	.6168	.6188	.6208	.6228	.6249	58°	4.0
32°	.6391	.6412	.6432	.6453	.6473	.6494	57°	4.1
33°	.6640	.6661	.8662	.6703	.6724	.6745	56°	4.2
34°	.6894	.6916	.6937	.6959	.6980	.7002	55°	4.3
35°	.7155	.7177	.7199	.7221	.7243	.7265	54°	4.4
36°	.7422	.7445	.7467	.7490	.7513	.7536	53°	4.5
37°	.7696	.7720	.7743	.7766	.7789	.7813	52°	4.7
38°	.7978	.8002	.8026	.8050	.8074	.8098	51°	4.8
39°	.8268	.8292	.8317	.8342	.8366	.8391	50°	4.9
40°	.8566	.8591	.8617	.8642	.8667	.8693	49°	5.1
41°	.8873	.8899	.8925	.8952	.8978	.9004	48°	5.2
42°	.9190	.9217	.9244	.9271	.9298	.9325	47°	5.4
43°	.9517	.9545	.9573	.9601	.9629	.9657	46°	5.6
44°	.9856	.9884	.9913	.9942	.9971	One	45°	5.8
	25'	20'	15'	10'	5'	0'	Cotan- gents.	

Subtract for cotangents.

Example.—To plat angle 56° 38':

$$\cot 56^\circ 35' = .6598$$

$$\text{d. } 4.2 \times 3 = 13$$

$$\cot 56^\circ 38' = .6585$$



TABLE OF FEET AND CHAINS.

(1 chain=100 links.)

Chains.	Feet.	Chains.	Feet.	Chains.	Feet.
.01	.66	.34	22.44	.67	44.22
.02	1.32	.35	23.10	.68	44.88
.03	1.98	.36	23.76	.69	45.54
.04	2.64	.37	24.42	.70	46.20
.05	3.30	.38	25.08		
.06	3.96	.39	25.74	.71	46.86
.07	4.62	.40	26.40	.72	47.52
.08	5.28			.73	48.18
.09	5.94	.41	27.06	.74	48.84
.10	6.60	.42	27.72	.75	49.50
		.43	28.38	.76	50.16
.11	7.26	.44	29.04	.77	50.82
.12	7.92	.45	29.70	.78	51.48
.13	8.58	.46	30.36	.79	52.14
.14	9.24	.47	31.02	.80	52.80
.15	9.90	.48	31.68		
.16	10.56	.49	32.34	.81	53.46
.17	11.22	.50	33.00	.82	54.12
.18	11.88			.83	54.78
.19	12.54	.51	33.66	.84	55.44
.20	13.20	.52	34.32	.85	56.10
		.53	34.98	.86	56.76
.21	13.86	.54	35.64	.87	57.42
.22	14.52	.55	36.30	.88	58.08
.23	15.18	.56	36.96	.89	58.74
.24	15.84	.57	37.62	.90	59.40
.25	16.50	.58	38.28		
.26	17.16	.59	38.94	.91	60.06
.27	17.82	.60	39.60	.92	60.72
.28	18.48			.93	61.38
.29	19.14	.61	40.26	.94	62.04
.30	19.80	.62	40.92	.95	62.70
		.63	41.58	.96	63.36
.31	20.46	.64	42.24	.97	64.02
.32	21.12	.65	42.90	.98	64.68
.33	21.78	.66	43.56	.99	65.34

Feet.	Chains.	Feet.	Chains.	Feet.	Chains.
1	.0151515	4	.0606061	7	.1060606
2	.0303030	5	.0757576	8	.1212121
3	.0454545	6	.0909091	9	.1363636

TRIGONOMETRIC FORMULAS.

Any Triangle (General Case).

Sides are proportional to sines of opposite angles. (1)

Sin ang opp given side: sin ang opp req side: : given side: req side. . (2)

Sum of sides: diff: : tang half sum other two angles: tang half diff. . (3)

Given: One side and two angles. Solve for "Req side" in (2).

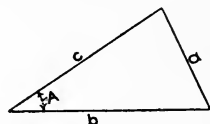
Given: Two sides and angle opposite one of them. Solve for "Req side" and for one of the angles, in (2).

Given: Two sides and included angle. Solve for "Tang half diff" in (3).

Given: Three sides. Solve for " $\sin \frac{1}{2}A$ " in the following:

$$\text{Let } s = \frac{(a+b+c)}{2}.$$

$$\text{Then, } \sin \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

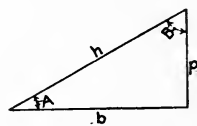


Right-angle Triangle (Special Case).

$$\sin A = \frac{p}{h}, \quad \cos A = \frac{b}{h}, \quad \text{Vers } A = \frac{h-b}{h}.$$

$$\tan A = \frac{p}{b}, \quad \cot A = \frac{b}{p}, \quad \text{Covers } A = \frac{h-p}{h}.$$

$$\sec A = \frac{h}{b}, \quad \text{Cosec } A = \frac{h}{p}, \quad \text{Exsec } A = \frac{h-b}{b}, \quad \text{Coexsec } A = \frac{h-p}{p}.$$



In which A = angle at base; b = base; p = perpendicular; h = hypotenuse; B = angle opposite base; $\sin A = \cos B$; etc.

NATURAL FUNCTIONS.

Radius r° curve = 5729.65 $2^\circ = 2864.93$ $3^\circ = 1910.08$ $4^\circ = 1432.69$ $5^\circ = 1146.28$ $6^\circ = 955.37$ $7^\circ = 819.02$ $8^\circ = 716.78$ $9^\circ = 637.27$ $10^\circ = 573.69$ 15° 30° 45° 60° 75°

Sine.

Tang.

Secant.

.25882

.50000

.70711

.86603

.96593

.26795

.57735

1.00000

1.73205

3.73205

1.03528

1.15470

1.41421

2.00000

3.86370

 $\pi = 3.141592 \quad \log \pi = 0.4971499$ 

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